DC CIRCUIT REVIEW

Internal Resistance. Batteries in a circuit generate *emf* (for "electromotive force"), written ε. The actual potential difference across terminals, called the *terminal voltage*, is less than the emf due to the *internal resistance* of the circuit, indicated by a lowercase r:

$$V_{\text{terminal}} = \varepsilon - Ir.$$

Resistors in Series. For resistors placed *in series*, or "in line," the equivalent resistance is:

$$R_{eq} = R_1 + R_2 + \dots.$$

Adding resistors in series is analogous to placing additional clogs in a pipe, decreasing the flow of current. **Resistors in Parallel.** For resistors placed *in parallel*, the equivalent resistance is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \dots.$$

Adding a resistor in parallel means creating another path for charges to flow through: $\downarrow R_{eq} \uparrow I$.

KIRKHOFF'S RULES

(1) *Junction Rule*: $\sum I_{in} = \sum I_{out}$. Current entering a node equals current leaving a node.

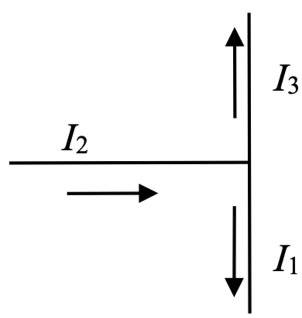
(2) *Loop Rule*: $\sum \Delta V = 0$. Sum of potential changes around a closed loop must equal zero.

DC Circuits are closed-loop paths in which conventional current flows from high to low potential. A **branch** is a portion of the circuit with shared current.

Voltmeters measure potential difference and have very high resistance. **Ammeters** measure current flow.

Using Kirkhoff's Rules.

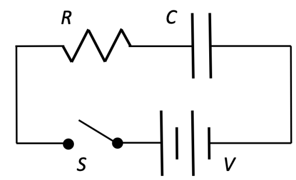
- Identify and label a *current* and *direction* of flow (with an arrow) for each branch of the circuit.
- Write out Node equations using the Junction Rule. Below, $I_2 = I_1 + I_3$.



- Write out Loop equations using the Loop rule. Pick a location in the loop, and trace in the direction of your (assumed) current until you return to the starting point. Going across the battery from low to high translates to $+V$; going across a resistor decreases potential by $-IR$.
- Solve the equations algebraically.

RC CIRCUITS

A *RC Circuit* has a resistor and capacitor in series.

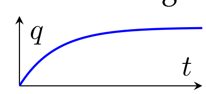


As current flows in the circuit, charge accumulates on the capacitor, creating a potential difference until $V_{\text{cap}} = V_{\text{battery}}$ and charge stops flowing.

Charge over time. Using $\sum \Delta V = 0$, we can derive charge q on the capacitor plates as a function of time:

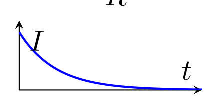
$$q(t) = CV_0(1 - e^{-\frac{t}{RC}}).$$

RC is called the *time constant* τ, the time to reach 0.632 of max charge.



Current over time. Taking the derivative of q, we obtain current:

$$I(t) = \frac{V_0}{R} e^{-\frac{t}{RC}}.$$



Discharging a capacitor. As charge drains from the plates, the potential difference across the plates (and the current) decreases, dissipating as heat. Using $\sum \Delta V = 0$,

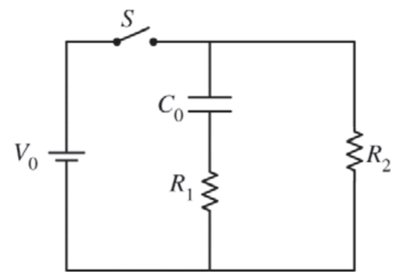
$$q(t) = CV_0 e^{\frac{-t}{RC}}.$$

PROBLEM 1 (2022 FRQ SET 2 QUESTION 2AB

A non-ideal capacitor has internal resistance that can be modeled as an ideal capacitor in series with a small resistor of resistance r_C. A group of students performs an experiment to determine the internal resistance of a capacitor. A circuit is to be constructed with the following available equipment: a single ideal battery of potential difference ΔV_0, a single ammeter, a single variable resistor of resistance R, a single uncharged non-ideal capacitor of capacitance C, and one or more switches as needed. Draw this circuit.

The capacitor is fully charged by the battery. At time $t = 0$, the capacitor starts discharging through the resistor. Show that the current I through the capacitor as a function of time t is $I(t) = I_0 e^{\frac{-t}{(R+r_C)C}}$ as the capacitor discharges.

PROBLEM 2 (2017 FRQ QUESTION 2ABDE)



In the circuit shown above, an ideal battery of voltage V_0 is connected to a capacitor with capacitance C_0 and resistors with resistances R_1 and R_2, with $R_1 > R_2$. The switch S is open, and the capacitor is initially uncharged.

(a) The switch is closed at time $t = 0$. Sketch the charge q on the capacitor as a function of time t. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

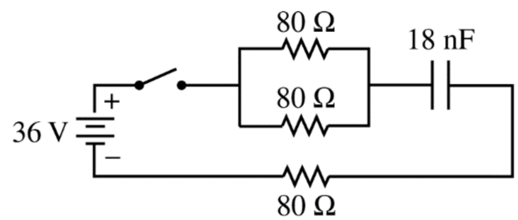
(b) Sketch the current I through each resistor as a function of time t. Clearly label the two curves as I_1 and I_2 , the currents through resistors R_1 and R_2, respectively. Explicitly label any intercepts, asymptotes, maxima, or minima with numerical values or algebraic expressions, as appropriate.

The circuit is constructed using an ideal $1.5V$ battery, an $80\mu F$ capacitor, and resistors $R_1 = 150\Omega$ and $R_2 = 100\Omega$. The switch is closed, allowing the capacitor to fully charge. The switch is then opened, allowing the capacitor to discharge.

(c) Calculate the current through resistor R_2 immediately after the switch is opened. Is the current through resistor R_2 increasing, decreasing, or constant immediately after the switch is opened?

(d) Calculate the energy stored in the capacitor immediately after the switch is opened. Then calculate the energy dissipated by resistor R_1 as the capacitor completely discharges.

PROBLEM 3 (2018 FRQ QUESTION 2DEF)



The student makes a capacitor using aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be $18nF$. The student uses this uncharged capacitor to build a circuit using wire, a $36V$ battery, 3 identical 80Ω resistors, and an open switch, as shown in the figure above.

(a) Calculate the current in the battery immediately after the switch is closed.

(b) Determine the time constant for this circuit.

(c) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured? Explain experimentally what could account for this.

ii. Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured? Explain experimentally what could account for this.

Question 2: Free-Response Question — 15 points

(a) For a schematic diagram with the capacitor in series with the resistor — **1 point**

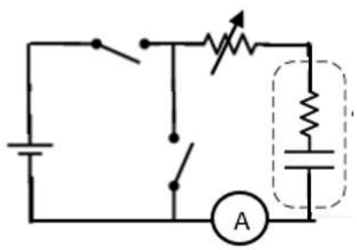
Scoring Note: The response may earn this point even if the variable resistor is not included in the circuit diagram.

For a schematic diagram with the ammeter in series with the capacitor and resistor — **1 point**

For a schematic diagram that uses a switch to connect the battery to the capacitor — **1 point**

For a schematic diagram that uses a switch that allows the capacitor to discharge through the resistor — **1 point**

Example Response



Total for part (a) — 4 points

(b) For using an appropriate loop equation by substituting a correct expression for the potential difference across the capacitor in terms of I, C, and r_C and IR for the potential difference across the variable resistor, if included — **1 point**

Example Response

$$V_C - V_R = 0$$

$$\frac{q}{C} - Ir_C = IR \therefore \frac{q}{C} = I(R + r_C)$$

For substituting $R + r_C$ as the total resistance of the circuit — **1 point**

Example Response

$$R_{\text{total}} = R + r_C$$

Scoring Notes:

- This point is earned if the above substitution is made anywhere in part (b).
- If the variable resistor is not included in the expression, accept expressions without R throughout.

For a correct differential equation consistent with the first point that could be used to determine the current I through the capacitor as a function of time t — **1 point**

Example Response

$$\frac{dq}{dt}\frac{1}{C} = (R + r_C)\frac{dI}{dt}$$
$$-I\frac{1}{C} = (R + r_C)\frac{dI}{dt}; \quad I = -\frac{dq}{dt}$$

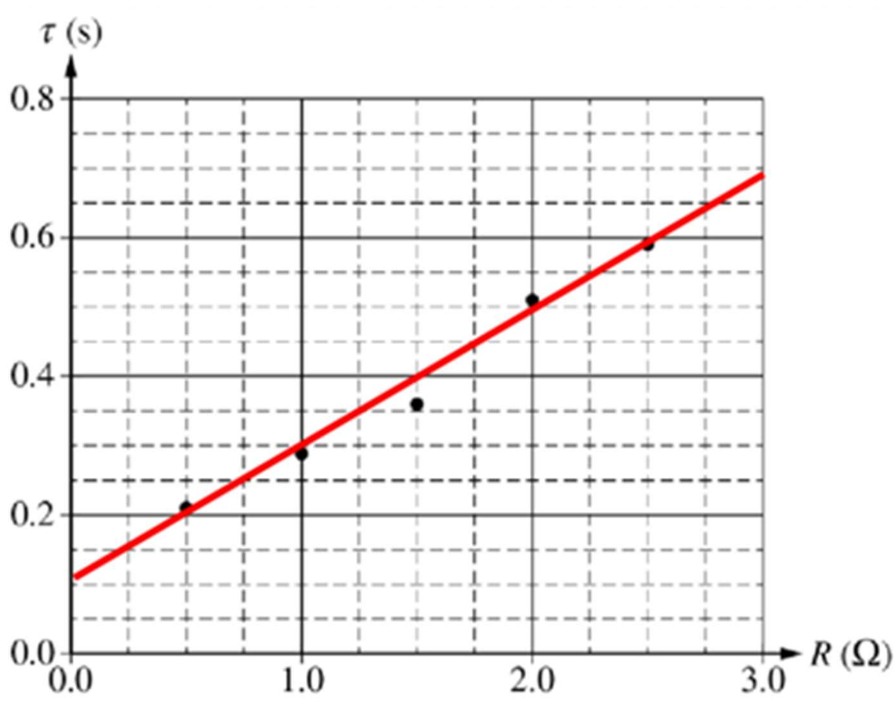
Where q is the charge on one plate of the capacitor that decreases over time

$$\frac{1}{I}\frac{dI}{dt} = \frac{-1}{C(R + r_C)}$$
$$\ln\frac{I}{I_0} = \frac{-t}{C(R + r_C)}$$
$$I = I_0 e^{\frac{-t}{C(R+r_C)}}$$

Total for part (b) 3 points

(c)(i) For drawing an appropriate best-fit line — **1 point**

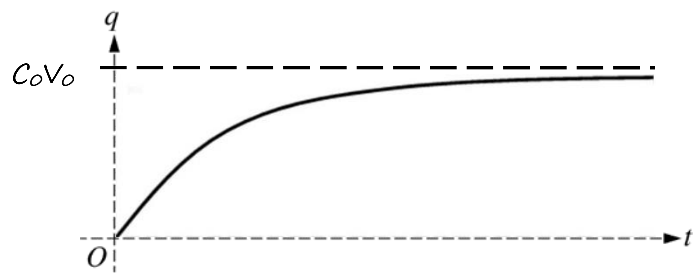
Example Response



Question 2

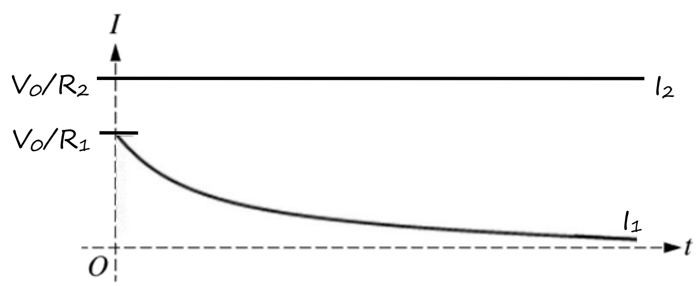
15 points total **Distribution of points**

(a) 3 points



For a concave down curve starting at the origin	1 point
For a horizontal asymptote	1 point
For properly labeling the horizontal asymptote	1 point

(b) 4 points



For showing that I_2 is horizontal with the correct vertical intercept	1 point
For showing that I_1 is concave up and asymptotic to I = 0	1 point
For correctly labeling the vertical intercept of the I_1 graph	1 point
For drawing the I_2 graph always above the I_1 graph	1 point

(c) 2 points

Select "$\Delta t_C < \Delta t_D$"

For indicating that the equivalent resistance during discharging is greater than during charging	1 point
For a statement relating the greater resistance to the greater time constant, or to a smaller current	1 point

Example: Because the resistance as the capacitor discharges is greater than when it charges, the time constant is larger for discharging. Therefore, the time to charge to 50% of its maximum charge is less than the time to discharge to 50% of its maximum charge.

Question 2 (continued)

Distribution of points

(d)

i. 2 points

For using a correct statement of Ohm's law or loop rule with the switch open — 1 point

$V = IR$

$I = \frac{V_C}{R_{tot}} = \frac{V_0}{R_1 + R_2}$

Substitute correct values into equation above

$I = \frac{(1.5\text{ V})}{(100\ \Omega + 150\ \Omega)}$

For a correct answer — 1 point

$I = 6.0\text{ mA}$ with units, or 0.006 without units

ii. 1 point

For a correct justification — 1 point

Example: Once the capacitor begins to discharge, the charge stored in the capacitor will decrease and the potential difference across the capacitor decreases. Therefore, the current through resistor R_2 decreases.

(e)

i. 1 point

For correctly substituting into an equation for the energy stored in a capacitor, including units for numerical values — 1 point

$U = \frac{1}{2}CV^2 = \frac{1}{2}(80\ \mu\text{F})(1.5\text{ V})^2 = 90\ \mu\text{J}$

ii. 2 points

For setting up a valid equation or argument to calculate the energy dissipated by resistor R_1 — 1 point

$E_{dis} = Pt = I^2Rt \therefore E_{dis} \sim R \therefore E_{dis,1} = E_{dis,tot}\frac{R_1}{(R_1 + R_2)}$

For correctly substituting into the equation above — 1 point

$E_{dis,1} = (90\ \mu\text{J})\frac{(150\ \Omega)}{(150\ \Omega + 100\ \Omega)} = 54\ \mu\text{J}$

Alternate Solution on next page.

Question 2 (continued)

Distribution of points

(e)(ii) continued

Alternate Solution — *Alternate Points*

Derive an equation for the current through resistor R_1 as a function of time

$$I(t) = I_{MAX}e^{-t/\tau} = \frac{V_{MAX}}{(R_1 + R_2)}e^{-t/(R_1+R_2)C} = \frac{(1.5\ \text{V})}{(250\ \Omega)}e^{-t/(250\ \Omega)(80\ \mu\text{F})}$$

$$I(t) = (6.0\ \text{mA})e^{-t/(0.02)}$$

For using a correct formula with integral calculus to calculate the energy dissipated by resistor R_1 — 1 point

$$E = \int Pdt = \int I^2Rdt = \int I(t)^2 R_1 dt = \int\left((6.0\ \text{mA})e^{-t/(0.02)}\right)^2 (150\ \Omega)dt$$

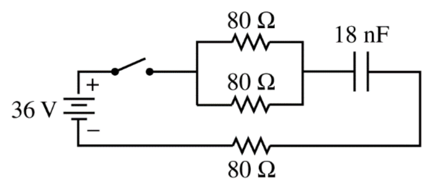
For integrating the above equation with correct limits or constants of integration — 1 point

$$E = (0.0054)\int_{t=0}^{t=\infty}\left(e^{-t/(0.02)}\right)^2 dt = (0.0054)\int_{t=0}^{t=\infty} e^{-100t}dt = (0.0054)\left[\frac{e^{-100t}}{(-100)}\right]_{t=0}^{t=\infty}$$

$$E = \left(-5.4\times 10^{-5}\right)\left(e^{-\infty} - e^{0}\right) = 54\ \mu\text{J}$$

Question 2 (continued)

Distribution of points



The student now makes a capacitor using the same aluminum foil plates and just one sheet of paper. Using the experimentally determined dielectric constant, the student calculates the capacitance to be 18 nF. The student uses this uncharged capacitor to build a circuit using wire, a 36 V battery, 3 identical 80 Ω resistors, and an open switch, as shown in the figure above.

(d) 3 points

Calculate the current in the battery immediately after the switch is closed.

For calculating the equivalent resistance for the parallel resistors		1 point
$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{80\ \Omega} + \frac{1}{80\ \Omega} = \frac{2}{80\ \Omega} = \frac{1}{40\ \Omega}$		
$R_P = 40\ \Omega$		
For using Ohm's law with the potential difference across the capacitor equal to zero		1 point
$I = \frac{V}{R} = \frac{V}{R_P + R_3} = \frac{(36\text{ V})}{R_P + R_3}$		
For substitution of values for resistance including the value for combined resistance above		1 point
$I = \frac{V}{R_P + R_3} = \frac{(36\text{ V})}{(40\ \Omega + 80\ \Omega)} = 0.30\text{ A}$		

(e) 2 points

Determine the time constant for this circuit.

For using the equation for the time constant with the equivalent resistance from above		1 point
$\tau = R_{eq}C = (120\ \Omega)(18\text{ nF})$		
For an answer with units consistent with part (d)		1 point
$t = 2.16\ \mu\text{s}$		

Question 2 (continued)

Distribution of points

(f) Students A and B measure the time it takes after the switch is closed for the voltage across the capacitor to reach half its maximum value and find that it is longer than expected.

i. 1 point

Student A assumes that the capacitance value is correct. Would Student A conclude that the resistance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
Example: The battery is not ideal and has internal resistance, so the actual resistance for the circuit is larger than the measured resistance.		

ii. 1 points

Student B assumes that the resistance value is correct. Would Student B conclude that the capacitance value is larger or smaller than measured?

____ Larger than measured ____ Smaller than measured

Explain experimentally what could account for this.

Select “Larger than measured”		
For an appropriate explanation		1 point
Example: Some of the sheets of paper may be thinner than expected, so the actual capacitance for the circuit is larger than the measured capacitance.		